Quiz 1 Solutions

Question 1

(1) $A_1 = \{vvv, vvd, vdv, vdd\}$

(2) $B_1 = \{dvv, dvd, ddv, ddd\}$

(3) $A_2 = \{vvv, vvd, dvv, dvd\}$

(4) $B_2 = \{vdv, vdd, ddv, ddd\}$

(5) $A_3 = \{vvv, ddd\}$

(6) $B_3 = \{vdv, dvd\}$

(7) $A_4 = \{vvv, vvd, vdv, dvv, vdd, dvd, ddv\}$

(8) $B_4 = \{ddd, ddv, dvd, vdd\}$

Recall that A_i and B_i are collectively exhaustive if $A_i \cup B_i = S$. Also, A_i and B_i are mutually exclusive if $A_i \cap B_i = \phi$. Since we have written down each pair A_i and B_i above, we can simply check for these properties.

The pair A_1 and B_1 are mutually exclusive and collectively exhaustive. The pair A_2 and B_2 are mutually exclusive and collectively exhaustive. The pair A_3 and B_3 are mutually exclusive but *not* collectively exhaustive. The pair A_4 and B_4 are not mutually exclusive since dvd belongs to A_4 and B_4. However, A_4 and B_4 are collectively exhaustive.

Question 2

There are exactly 50 equally likely outcomes: s_{51} through s_{100}. Each of these outcomes has probability 0.02.

(1) $P[\{s_{79}\}] = 0.02$

(2) $P[\{s_{100}\}] = 0.02$

(3) $P[A] = P[\{s_{90}, \ldots, s_{100}\}] = 11 \times 0.02 = 0.22$

(4) $P[F] = P[\{s_{51}, \ldots, s_{59}\}] = 9 \times 0.02 = 0.18$

(5) $P[T \geq 80] = P[\{s_{80}, \ldots, s_{100}\}] = 21 \times 0.02 = 0.42$

(6) $P[T < 90] = P[\{s_{51}, s_{52}, \ldots, s_{89}\}] = 39 \times 0.02 = 0.78$

(7) $P[\text{a } C \text{ grade or better}] = P[\{s_{70}, \ldots, s_{100}\}] = 31 \times 0.02 = 0.62$

(8) $P[\text{student passes}] = P[\{s_{60}, \ldots, s_{100}\}] = 41 \times 0.02 = 0.82$

Question 3

We can describe this experiment by the event space consisting of the four possible events VB, VL, DB, and DL. We represent these events in the table:

	V	D
L	0.35	?
B	?	?

In a roundabout way, the problem statement tells us how to fill in the table. In particular,

$$P[V] = 0.7 = P[VL] + P[VB] \qquad (1)$$

$$P[L] = 0.6 = P[VL] + P[DL] \qquad (2)$$

Since $P[VL] = 0.35$, we can conclude that $P[VB] = 0.35$ and that $P[DL] = 0.6 - 0.35 = 0.25$. This allows us to fill in two more table entries:

	V	D
L	0.35	0.25
B	0.35	?

The remaining table entry is filled in by observing that the probabilities must sum to 1. This implies $P[DB] = 0.05$ and the complete table is

	V	D
L	0.35	0.25
B	0.35	0.05

Finding the various probabilities is now straightforward:

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(1) $P[DL] = 0.25$

(2) $P[D \cup L] = P[VL] + P[DL] + P[DB] = 0.35 + 0.25 + 0.05 = 0.65$.

(3) $P[VB] = 0.35$

(4) $P[V \cup L] = P[V] + P[L] - P[VL] = 0.7 + 0.6 - 0.35 = 0.95$

(5) $P[V \cup D] = P[S] = 1$

(6) $P[LB] = P[LL^c] = 0$
